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Research Article

Application of partial sliding mode in guidance problem

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ABSTRACT

In this paper, the problem of 3-dimensional guidance law design is considered and a new guidance law based on partial sliding mode technique is presented. The approach is based on the classification of the state variables within the guidance system dynamics with respect to their required stabilization properties. In the proposed law by using a partial sliding mode technique, only trajectories of a part of states variables are forced to reach the partial sliding surfaces and slide on them. The resulting guidance law enables the missile to intercept highly maneuvering targets within a finite interception time. Effectiveness of the proposed guidance law is demonstrated through analysis and simulations.

1. Introduction

Design of guidance laws for intercepting of highly maneuvering targets is a difficult nonlinear problem. The missile must intercept its maneuvering target within a finite interception time, in some case, only in a few seconds. Recently, nonlinear control theories have been used in design of robust guidance laws. Laws such as: Lyapunov-based nonlinear guidance laws [1–3], first-order sliding mode guidance laws [4–8], and nonlinear H_∞ guidance laws [9–11], are obtained based on Lyapunov theorems on asymptotic stability or exponential stability of all states. In [12,13], guidance laws are obtained by applying a second-order sliding-mode control method. However, these laws, because of complexity in the structure, only have been applied to planner interception geometry.

It was shown in [14] that in a practical situation, asymptotic stability behavior is not a realistic behavior for all states of the guidance system and a new guidance law based on partial stability principle was designed for planner interception geometry. The approach was based on the classification of the state variables of the guidance system into two parts with respect to their required stabilization properties where stability is desired behavior for only the first part of state variables.

In this paper, a new 3-dimensional missile guidance law based on *Partial Sliding Mode technique* (PSM) is designed. The PSM technique was first introduced in [15] and in this work it is applied to the guidance control problem.

Most of the existing methods in the field of nonlinear control theory are based on stabilization of all states and in spite of applications of partial stability in many of engineering fields [16], there are a few papers regarding the design of control laws which stabilize only a part of system's states. Also, among the existing papers in the field of partial control, most of them only consider a case study and try to design control laws for partial stability of their specific applications [17–19] and do not pose a general framework in designing partial control laws. It is worth noting that the control schemes posed in Refs. [17,20–22] are uneasy to realize and are usable only for systems with some special structures. The PSM technique leads to design of a partial stabilizing controller for a nonlinear system in the presence of uncertainties and external disturbances. In this method partial sliding surfaces are designed such that restricting the motion on these surfaces guarantees asymptotic stabilization of only the first part of state variables. This is in contrast to standard sliding mode (SM) technique that stabilizes all state variables by forcing them to reach sliding manifolds (which are functions of all states) and slide on them toward the equilibrium point. Another structural difference between PSM and SM is that in PSM, the reduced input vector; was designed while in the SM, the input vector is *wholly* designed. This idea makes the possibility of converting the control problem into a simpler one having fewer control input variables. Additionally, the remaining control variables make the possibility of imposing some constraints on the behavior of the other state variables [15].

In the present work, the PSM method was applied in guidance law design. This approach is advantageous from practical view-points,

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since it forces the states of the guidance system to behave as it is desirable in practice and is compatible with a successful guidance scenario. In addition, the acceleration vector of the target is assumed as an external bounded disturbance and only its bound is required in the design of the guidance law and the accurate measurement of target acceleration during the flight is not necessary. Effectiveness of the designed guidance law in achieving zero-miss-distance within a finite interception time is demonstrated analytically and also through computer simulations.

2. Problem definition

2.1. Missile/target kinematics model

In this section, a 3-dimensional air to air engagement is considered. The vehicles are modeled as point masses. The relative motion between the missile and the target is described in the spherical coordinate system (r,θ,ϕ) with the origin fixed on the center of the mass of the missile. The 3-dimensional missile–target geometry is shown in Fig. 1, where r is the relative distance between the missile and the target. Also, θ and ϕ are yaw and pitch line of sight angles, respectively. Let (e_r,e_θ,e_ϕ) be unit vector along the coordinate axes and $\vec{r}=r\vec{e}_r$ be the relative distance vector along the line of sight (LOS) vector (i.e., the vector connecting the missile to the target at each moment with direction toward the target).

The differentiation of $\vec{r}$ gives the 3-dimensional relative velocity as

$$\dot{\vec{r}}=\dot{r}\vec{e}_r+r\dot{\theta}\cos\phi\,\vec{e}_\theta+r\dot{\phi}\vec{e}_\phi \tag{1}$$

where $\dot{r}$ is the radial component of relative speed. Also, $r\dot{\theta}\cos\phi$ and $r\dot{\phi}$ are tangential components of relative speed between the missile and the target, respectively. Differentiating Eq. (1) yields the components of the relative acceleration as

$$\begin{cases} \ddot{r}-r\dot{\phi}^2-r\dot{\theta}^2\cos^2\phi=\omega_r-u_r \\ r\ddot{\theta}\cos\phi+2\dot{r}\dot{\theta}\cos\phi-2r\dot{\phi}\dot{\theta}\sin\phi=\omega_\theta-u_\theta \\ r\ddot{\phi}+2\dot{r}\dot{\phi}+r\dot{\theta}^2\cos\phi\sin\phi=\omega_\phi-u_\phi \end{cases} \tag{2}$$

where $\omega=(\omega_r,\omega_\theta,\omega_\phi)^T$ and $u=(u_r,u_\theta,u_\phi)^T$ are the acceleration vectors of target and missile, respectively. Here, acceleration of the target is assumed as an external bounded disturbance. It is worth noting that only this bound is required in the design process of guidance law and the accurate measurement of target acceleration during the maneuver is not necessary. In the sequel, we consider $x(t)=[\theta(t),\phi(t),\dot{\theta}(t),\dot{\phi}(t),r(t),\dot{r}(t)]^T$ as the state vector.

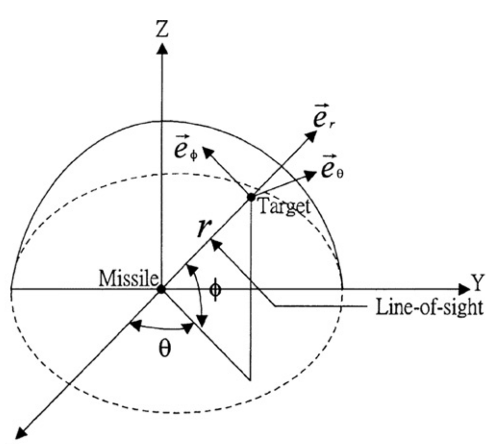

Fig. 1. 3-dimensional missile–target geometry.

Note: Usually, the initial conditions of engagement are such that $r_0>0$ and $\dot{r}_0<0$ (as assumed in the present research). It means that the target is in front of the missile and the missile is approaching it.

2.2. Desirable behaviors for each state variable

For the interception, it is sufficient that $r(t)$ becomes zero in an instance, say t_f, where t_f is the interception time, i.e., $r(t_f)=0$ (zero-miss-distance). Clearly, it is not convenient for $r(t)$ to asymptotically converge to zero. In other words, the asymptotic convergence is not an ideal behavior for $r(t)$. It should be noted that asymptotic convergence behavior means that the missile approaches the target within an infinite time interval. It is evident that such a behavior is not a desirable behavior in missile guidance, which is supposed to hit the target with a nonzero speed in a finite time. For this purpose, it is sufficient that the relative radial speed between the missile and the target satisfies the following proposition.

Proposition 1. *In order to intercept the target within a finite time interval* ($r(t_f)=0$), *it is sufficient to have*:

$$\exists t_1\in[t_0\ \ t_f)\quad \text{s.t. } \dot{r}\le-\zeta<0\quad \forall t\in[t_1\ \ t_f] \tag{3}$$

where ζ is a positive constant. Also, $r(t)$ is positive and continuous. Consequently, to reach a zero value, i.e. zero-miss-distance, within a finite time, it should decrease from its value at $t_1(<t_f)$ down to a zero value at t_f. Inequality (3) implies $r(t)-r(t_1)\le-\zeta(t-t_1)$, which shows that $r(t)$ reduces within the time interval $[t_1\ \ t]$. Since it is desirable to have $r(t_f)=0$, thus this leads to

$$t_f\le\frac{r(t_1)+\zeta t_1}{\zeta} \tag{4}$$

Hence, our expectation of $\dot{r}$ is that it satisfies Proposition 1. In other words, convergence and stability of this state is not under consideration. Although regulating $\dot{r}$ to a negative constant c guarantees interception of the non-maneuvering target within a finite time, it is not efficient for interception of a highly maneuvering target in an acceptable interception time. In such a case, to improve the performance, c should be time varying; however, for its determination, the maneuvering model of the target should be known for the missile, while it is not always possible in practice [14].

Regarding the other state variables, i.e., θ, $\dot{\theta}$, ϕ, $\dot{\phi}$, in contrast to two first states, i.e., r and $\dot{r}$, asymptotic convergence behavior is desirable. An appropriate guidance law, in addition to decreasing the relative distance, must keep the pitch and yaw LOS-angular rates as small as possible [11]. It means that it is desirable to have $(\dot{\theta},\dot{\phi})\to(0,0)$, $\theta\to c_1$ and $\phi\to c_2$, where c_1 and c_2 may be free or some pre-specified constants. Therefore, the state vector may be separated into $x_1=[\theta\ \phi\ \dot{\theta}\ \dot{\phi}]^T$ and $x_2=[r\ \dot{r}]^T$ where the asymptotic stability behavior is desirable only for x_1.

3. Partial sliding mode control method

In this section, a robust partial stabilization technique for a class of nonlinear dynamical systems based on the first order sliding mode control idea is presented [15]. This partial control method makes the possibility of converting the control problem into a simpler one having fewer control input variables while in other existing papers in the field of partial control, the input vector is *wholly* designed. For this purpose, the state vector ($x\in R^n$) is separated into two parts: $x_1\in R^{n_1}$ and $x_2\in R^{n_2}$, $(n_1+n_2=n)$ and accordingly the nonlinear dynamical system is divided into two subsystems. The subsystems, hereafter, are referred as $\dot{x}_1$-subsystem and $\dot{x}_2$-subsystem. The reduced control input vector (the vector that includes components of input vector appearing in the $\dot{x}_1$-subsystem) is designed based on the new sliding mode control

technique, i.e., "*partial sliding mode*" in such a way to guarantee asymptotic stability of the nonlinear system with respect to the first part of states vector, i.e., x_1 for every x_2. Furthermore, the remaining control variables (which do not appear in the $\dot{x}_1$-subsystem but appear in the $\dot{x}_2$-subsystem) make this possibility to impose some constraints on the behavior of the second part of states (x_2). In brief, there are two main structural differences between partial sliding mode and the first order sliding mode techniques. First, in partial sliding mode, sliding surfaces is a function of only x_1; while in the conventional first order sliding mode technique, the sliding surfaces is a function of x. Second, in partial sliding mode method, the reduced input vector is designed while in the conventional one, the input vector is wholly designed. Therefore, partial sliding mode control results in the stability of x_1 for every x_2, while the conventional technique results in the stability of all states, i.e., x.

Consider the following nonlinear control system [15]:

$$\dot{x} = f(x,u), \quad x(t_0) = x_0 \tag{5}$$

where $x \in R^n$ is the state vector and $u \in R^m$ is the input vector. Let vectors x_1 and x_2 denote the partitions of the state vector, respectively. Therefore, $x = (x_1^T, x_2^T)^T$ where $x_1 \in R^{n_1}$, $x_2 \in R^{n_2}$ and $n_1 + n_2 = n$. As a result, the nonlinear system (5) can be divided into two parts:

$$\begin{aligned} \dot{x}_1(t) &= F_1(x_1(t), x_2(t), u), \quad x_1(t_0) = x_{10} \\ \dot{x}_2(t) &= F_2(x_1(t), x_2(t), u), \quad x_2(t_0) = x_{20} \end{aligned} \tag{6}$$

Definition 1. The nonlinear control system (6) is said to be asymptotically stabilizable with respect to x_1 for every $x_2 \in R^{n_2}$, if there exists some admissible feedback control law $u = k(x_1, x_2)$, which makes the nonlinear system (6) asymptotically stable with respect to x_1. It means that in the closed-loop system for every $\varepsilon > 0$ and $x_{20} \in R^{n_2}$, there exists $\delta = \delta(x_{20}) > 0$ such that $||x_{10}|| < \delta$ implies $||x_1(t)|| < \varepsilon$ for all $t \geq 0$ and also $\lim_{t \to \infty} x_1(t) = 0$.

Now, suppose the $\dot{x}_1$-subsystem in Eq. (6) is affine with respect to input (the $\dot{x}_2$-subsystem may have the general dynamical form). Therefore,

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) + \sum_{i=1}^{m} g_{1i}(x_1, x_2) u_i \\ \dot{x}_2(t) &= F_2(x_1, x_2, u) \end{aligned} \tag{7}$$

where u_i is the ith component of input vector u. Also, $g_{1i} \in R^{n_1}$, for $i = 1, 2, \ldots, m$. Let k be the number of g_{1i} vectors which are nonzero. Hence, k is the number of components of input vector which appear in $\dot{x}_1$-subsystem and thus $0 \leq k \leq m$. Now, we assume that $k \neq 0$. By augmenting the k nonzero vectors g_{1i} in a matrix, the nonlinear system (7) can be rewritten as follows:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) + G_1(x_1, x_2)\overline{u} \\ \dot{x}_2 &= F_2(x_1, x_2, u) \end{aligned} \tag{8}$$

where $\overline{u} \in R^k$ is the reduced version of input vector u, that contains k control variables appearing in $\dot{x}_1$-subsystem, $G_1(x_1, x_2)$ is a $n_1 \times k$ matrix where its columns are the k nonzero vectors g_{1i}. In this case, the task is to find an appropriate $\overline{u}$, which guarantees partial stabilization of nonlinear system (8) with respect to x_1 by PSM technique.

Let $Z : D \to R^{n_1}$ be a diffeomorphism that transforms only $\dot{x}_1$-subsystem in (8) into the regular form. Thus,

$$\begin{cases} \begin{cases} \dot{\eta} = f_{a1}(\eta, \xi, x_2) \\ \dot{\xi} = f_{b1}(\eta, \xi, x_2) + g_{b1}(x_1, x_2)\overline{u} \end{cases} \\ \dot{x}_2(t) = F_2(x_1, x_2, u) \end{cases} \tag{9}$$

where $[\eta^T \ \xi^T]^T = Z(x_1)$, $\xi, \overline{u} \in R^k$ and $\eta \in R^{n_1 - k}$. Suppose $g_{b1}(x_1, x_2) = G_k(x_1, x_2) E_k(x_1, x_2)$, where $E_k(x_1, x_2) \in R^{k \times k}$ is a nonsingular matrix and $G_k(x_1, x_2) \in R^{k \times k}$ is a diagonal matrix whose elements are positive and greater than a nonzero lower bound. Under this assumption, the partial sliding manifold is selected as $s = \xi - \varphi(\eta) = 0$, where $\varphi(\eta)$ is a continuously differentiable function with $\varphi(0) = 0$. In this case, the sliding manifold is a function of only the first part of states, i.e., ξ and η. It was shown in [15] that if $\varphi(\eta)$ designed such that:

$$\frac{\partial V(\eta)}{\partial \eta} f_{a1}(\eta, \varphi(\eta), x_2) \leq -\gamma(||\eta||) \quad \forall (\eta, x_2) \in R^{n_1 - k} \times R^{n_2} \tag{10}$$

where $V : R^{n_1 - k} \to R$ is a positive definite, continuously differentiable function, and $\gamma(.)$ is a class K function, then the reduced-order model of $\dot{x}_1$-subsystem, i.e., $\dot{\eta} = f_{a1}(\eta, \varphi(\eta), x_2)$ is asymptotically stable with respect to η for all $x_2 \in R^{n_2}$ and by using the feedback control law (11), only trajectories of a part of state variables (i.e., η and ξ) are forced to reach the partial sliding manifold ($s = \xi - \varphi(\eta) = 0$) and stay on it.

$$\overline{u} = (G_k E_k)^{-1} \left(\frac{\partial \varphi}{\partial \eta} f_{a1} - f_{b1} \right) \tag{11}$$

Now, consider the system (9) in the presence of uncertainties and disturbances:

$$\begin{cases} \begin{cases} \dot{\eta} = f_{a1}(\eta, \xi, x_2) \\ \dot{\xi} = f_{b1}(\eta, \xi, x_2) + g_{b1}(x_1, x_2)\overline{u} + h_1(x_1, x_2, t) \end{cases} \\ \dot{x}_2(t) = F_2(x_1, x_2, u) + h_2(x_1, x_2, t) \end{cases} \tag{12}$$

where h_1 and h_2 are unknown nonlinear vector functions which satisfy the matching conditions; that is, the uncertain terms enter the state equations at the same point as the control input. These terms could result from modeling error and/or uncertainties and disturbances, which exists in any practical problem. In a typical situation, h_1 and h_2 are unknown, but some information about them, like an upper bound on their norm may be known. Suppose $h_1(x_1, x_2, t) = B_k(x_1, x_2, t)\overline{\varpi}(x_1, x_2, t)$, where $B_k \in R^{k \times k}$ is a diagonal matrix and $\overline{\varpi} \in R^{k \times 1}$ may be an unknown vector with a known upper bound $\overline{\gamma}(x_1, x_2, t)$, i.e.,

$$|\overline{\varpi}_i(x_1, x_2, t)| < \overline{\gamma}_i(x_1, x_2, t) \quad \text{for } i = 1, \ldots, k \tag{13}$$

In this case, the task is to design $\overline{u}$ to force s toward the manifold $s = 0$. It was shown in [15] that if ratio B_{ki}/G_{ki} satisfies the below inequality for $i = 1, \ldots, k$,

$$\left| \frac{B_{ki}(x_1, x_2, t)}{G_{ki}(x_1, x_2)} \right| \leq \rho_i(x_1, x_2) \tag{14}$$

where $\rho_i(x_1, x_2)$ is a known positive continuous function and B_{ki} and G_{ki} are the ith elements of diagonal matrixes B_k and G_k on its diagonal. Then the following feedback law guarantees robust partial stabilization:

$$\overline{u} = E_k^{-1} \left\{ G_k^{-1} \left(\frac{\partial \varphi}{\partial \eta} f_{a1} - f_{b1} \right) + \overline{v} \right\} \tag{15}$$

where $\overline{v} \in R^k$ and its ith component is

$$\overline{v}_i = -\beta_i(x_1, x_2) \operatorname{sgn}(s_i) \tag{16}$$

and $\beta_i(x_1, x_2) \geq \rho_i(x_1, x_2)\overline{\gamma}_i(x_1, x_2, t) + \beta_{0i}$ where β_{0i} is a positive constant. Using (15) guarantees that all trajectories starting off the manifold $s = 0$ reach it in a finite time and those on the manifold cannot leave it. Restricting the motion on this manifold guarantees stability with respect to ξ and η (partial stability) for all $x_2 \in R^{n_2}$ [15].

Remark 1. For the case $0 < k < m$, there are $l = m - k$ components of control input vector u not appearing in the $\dot{x}_1$-subsystem and therefore do not appear in $\overline{u}$, which appear in the $\dot{x}_2$-subsystem. By putting these components in vector $\tilde{u} \in R^l$, this vector may be chosen in a way that the response of x_2 satisfies the constraints related to practical considerations of the application. If there is an uncertain term $h_2(x_1, x_2, t)$ in the $\dot{x}_2$-subsystem, the additional term $\tilde{v}$ could be

designed in a way that the control law $\tilde{u}+\tilde{v}$, guarantees the desirable behavior for x_2 in the presence of $h_2(x_1,x_2,t)$ (as illustrated in the next section).

Remark 2. Since discontinuous controllers suffer from chattering, one way to alleviate this problem is to consider an approximation of the signum function by a saturation function with a high slope.

4. Guidance law design

In this section, we employ the PSM technique to fulfill the design task of the guidance law.

4.1. PSM guidance law

Let $x_1=[\eta^T\ \xi^T]^T$, where $\eta=[\eta_1\ \eta_2]^T=[\theta\ \phi]^T$ and $\xi=[\xi_1\ \xi_2]^T=[\dot{\theta}\ \dot{\phi}]^T$. Also, $x_2=[x_{21}\ x_{22}]=[r\ \dot{r}]^T$. $\omega=(\omega_r,\omega_\theta,\omega_\phi)^T$ and $u=(u_r,u_\theta,u_\phi)^T$ (the acceleration vectors of target and missile), are assumed as external disturbance and input control vector, respectively. The kinematics equations (2) can be rewritten in the following nonlinear state-space equation:

$$\dot{x}_1\text{-subsystem}:\begin{cases}\dot{\eta}_1=\xi_1\\ \dot{\eta}_2=\xi_2\\ \dot{\xi}_1=2\xi_1\xi_2\tan\eta_2-2\frac{x_{22}\xi_1}{x_{21}}-\frac{1}{x_{21}\cos\eta_2}u_\theta+\frac{1}{x_{21}\cos\eta_2}\omega_\theta\\ \dot{\xi}_2=-2\frac{x_{22}\xi_2}{x_{21}}-\xi_1^2\cos\eta_2\sin\eta_2-\frac{1}{x_{21}}u_\phi+\frac{1}{x_{21}}\omega_\phi\end{cases}$$

$$\dot{x}_2\text{-subsystem}:\begin{cases}\dot{x}_{21}=x_{22}\\ \dot{x}_{22}=x_{21}\xi_2^2+x_{21}\xi_1^2\cos^2\eta_2-u_r+\omega_r\end{cases}\tag{17}$$

Note that, the $\dot{x}_1$-subsystem in dynamical system (17) is already in the regular form. By substituting (17) in the structure (12), one may define:

$$f_{a1}=\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix},\quad f_{b1}=\begin{bmatrix}2\xi_1\xi_2\tan\eta_2-2\frac{x_{22}\xi_1}{x_{21}}\\ -2\frac{x_{22}\xi_2}{x_{21}}-\xi_1^2\cos\eta_2\sin\eta_2\end{bmatrix}\tag{18a}$$

$$g_{b1}=\begin{bmatrix}-\frac{1}{x_{21}\cos\eta_2} & 0\\ 0 & -\frac{1}{x_{21}}\end{bmatrix}=\underbrace{\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}}_{E_k}\underbrace{\begin{bmatrix}\frac{1}{x_{21}\cos\eta_2} & 0\\ 0 & \frac{1}{x_{21}}\end{bmatrix}}_{G_k};\bar{u}=\begin{bmatrix}u_\theta\\ u_\phi\end{bmatrix}\tag{18b}$$

$$h_1(x_1,x_2,t)=\begin{bmatrix}\frac{1}{x_{21}\cos\eta_2}\omega_\theta\\ \frac{1}{x_{21}}\omega_\phi\end{bmatrix}=\underbrace{\begin{bmatrix}\frac{1}{x_{21}\cos\eta_2} & 0\\ 0 & \frac{1}{x_{21}}\end{bmatrix}}_{B_k}\underbrace{\begin{bmatrix}\omega_\theta\\ \omega_\phi\end{bmatrix}}_{\overline{\omega}}\tag{18c}$$

and

$$F_2(x_1,x_2,u)=\begin{bmatrix}x_{22}\\ x_{21}\xi_2^2+x_{21}\xi_1^2\cos^2\eta_2-u_r\end{bmatrix},\quad h_2(x_1,x_2,t)=\begin{bmatrix}0\\ \omega_r\end{bmatrix}=\begin{bmatrix}0\\ 1\end{bmatrix}\omega_r\tag{18.d}$$

It is worth noting that throughout the interception process, it is logical to assume that $x_{21}\triangleq r>0$ and $|\phi|<\pi/2$ [5]. Also, x_{21} is smaller than its initial value, i.e., $x_{21}(0)=r_0$. This implies that $G_{ri}>1/r_0>0$ for $i=1,2,\ldots$. To fulfill the PSM technique, we choose the partial sliding surface as

$$s=(s_1,s_2)^T=\xi+M\eta\tag{19}$$

where $\varphi(\eta)=-M\eta$ and $M\in R^{2\times 2}$ is a positive definite matrix. It is worth nothing that if $\xi-\eta$ lies on the partial sliding surface, the reduced model have the form:

$$\dot{\eta}+M\eta=0\tag{20}$$

which implies exponential decaying of η. Furthermore, according to Eq. (15):

$$\begin{aligned}\bar{u}&=E_k^{-1}\left\{\left(\frac{\partial\varphi}{\partial\eta}f_{a1}-f_{b1}\right)+\overline{v}\right\}\\ &=-G_k^{-1}(-M\xi-f_{b1})-\overline{v}\end{aligned}\tag{21}$$

where E_k is substituted from (18b). Also, f_{b1} is given in (18a). The final step is to construct an extra control $\overline{v}$ that can compensate for the effect of target acceleration vector (external disturbance). For this purpose, assume that the components of acceleration vector of the target satisfy the following assumption:

Assumption 1. There exists a vector function.

$$\overline{\gamma}(x_1,x_2)=[\overline{\gamma}_1,\overline{\gamma}_2]^T=[\gamma_\theta(x_1,x_2),\gamma_\phi(x_1,x_2)]^T\tag{22}$$

where γ_θ and γ_ϕ are non-negative functions such that $|\overline{\omega}_i|\le\overline{\gamma}_i$ for $i=1,2$.

Furthermore, $|B_{ki}/G_{ki}|=1$ for $i=1,2$ (refer to (18b) and (18c)). Therefore, $\rho_i(x_1,x_2)=1$. Now, by considering,

$$\overline{v}_i=-\beta_i(x_1,x_2)\mathrm{sgn}(s_i)\tag{23}$$

where

$$\begin{aligned}\beta_i(x_1,x_2)&\ge\rho_i(x_1,x_2)\overline{\gamma}_i(x_1,x_2)+\beta_{0i}\\ &=\overline{\gamma}_i(x_1,x_2)+\beta_{0i}\end{aligned}\tag{24}$$

As a result, the reduced control vector (21) achieves the desired performance for x_1. Now, in order to complete the design process, $\tilde{u}=u_r$ should be obtained to attain the appropriate behavior of x_2 (refer to Remark 1). Assume $|\omega_r|\le\gamma_r(x_1,x_2)$ and take

$$u_r=x_{21}\xi_2^2+x_{21}\xi_1^2\cos^2\eta_2-\sigma x_{22}+v_r\tag{25}$$

where $\sigma>0$ and $v_r=\gamma_r(x_1,x_2)$. By substituting (25) in (17), one has

$$\dot{x}_{22}=\sigma x_{22}-\gamma_r+\omega_r$$

Thus,

$$\begin{aligned}x_{22}(t)&=x_{22}(0)e^{\sigma t}+\int_0^t(-\gamma_r+\omega_r)e^{\sigma(t-\tau)}\,d\tau\\ &\le x_{22}(0)e^{\sigma t}-\int_0^t\gamma_r e^{\sigma(t-\tau)}\,d\tau+\int_0^t\gamma_r e^{\sigma(t-\tau)}\,d\tau\end{aligned}\tag{26}$$

Consequently, $x_{22}(t)\le x_{22}(0)<0$. Therefore, by choosing $-\zeta=x_{22}(0)$, Proposition 1 is satisfied and the desirable behavior for x_2 is achieved. In addition, the relative distance will turn into zero within a finite time. Clearly, higher values of σ makes a shorter time of interception; however, its adjustment should be done with respect to physical limitations [14].

Consequently, the PSM guidance law (27) guarantees intercepting of maneuvering targets within a finite interception and zero-miss distance.

$$\begin{cases}\bar{u}=\begin{bmatrix}u_\theta\\ u_\phi\end{bmatrix}=-G_k^{-1}(-M\xi-f_{b1})-\overline{v}\\ u_r=x_{21}\xi_2^2+x_{21}\xi_1^2\cos^2\eta_2-\sigma x_{22}+\gamma_r\end{cases}\tag{27}$$

where $\xi=[\xi_1\ \xi_2]^T=[\dot{\theta}\ \dot{\phi}]^T$, $\eta=[\eta_1\ \eta_2]^T=[\theta\ \phi]^T$, $s=(s_1,s_2)^T=\xi+M\eta$, ($M\in R^{2\times 2}$ is a positive definite matrix), $x_2=[x_{21}\ x_{22}]=[r\ \dot{r}]^T$, and $\overline{v}=[\overline{v}_1,\overline{v}_2]^T$ where $\overline{v}_i=-\beta_i(x_1,x_2)\mathrm{sgn}(s_i)$, for $i=1,2$. Also, G_k and f_{b_1} are given in Eqs. (18a) and (18b), respectively.

4.2. Computer simulations

Numerical simulations are performed to illustrate the effectiveness of the PSM guidance law against a highly maneuvering

target. The target is assumed maneuvering at the following trajectory:

$$\omega(t) = 70\sin(0.5t)\vec{e}_r + 70\sin(0.5t+\pi/4)\vec{e}_\theta + 70\cos(0.5t)\vec{e}_\phi \tag{28}$$

The initial states are chosen as $r_0 = 5$ km, $\theta_0 = \pi/3$, $\phi_0 = \pi/3$, $\dot{\theta} = 0.08$ rad/s, $\dot{\phi} = 0.06$ rad/s and $\dot{r} = -300$ m/s. The initial conditions in the Inertia-frame of the missile and the target are given by

- For the missile: $x_M(0) = y_M(0) = 0$ m
- For the target: $x_T(0) = 5000\cos\phi_0\cos\theta_0 m$, $y_T(0) = 5000\cos\phi_0\sin\theta_0 m$, $z_T(0) = 5000\sin\phi_0 m$ and $v_{Tx}(0) = v_{Ty}(0) = v_{Tz}(0) = 100$ m/s.

In addition, $\beta_{01} = \beta_{02} = 0.1$, $\sigma = 0.03$ and the three non-negative functions $\gamma_r(x_1,x_2)$, $\gamma_\theta(x_1,x_2)$ and $\gamma_\phi(x_1,x_2)$ are assumed as $\gamma_r = \gamma_\theta = \gamma_\phi = 70$. Also, $M = diag\{2.41, 2.41\}$ is selected. In the meantime, for the purpose of attention of chattering, instead of $sgn(s_i)$, the saturation function $sat(s_i/\varepsilon)$ with $\varepsilon = 1$, is used.

The PSM guidance law is compared with the SM guidance law presented in [5]. It is because that the same state space model has been used in [5]. Additionally SM guidance law has been compared with some existing guidance laws in [5] and has achieved better results. It is shown in this section that the proposed guidance law has gained the better performance in terms of interception time and control effort, compared to the SM guidance law. In SM guidance law stability of all states is under consideration in contrast to the PSM guidance law. Numerical simulations are given in Figs. 2–4. As seen, the PSM law has totally gained the better performance in terms of interception time (t_f) and control efforts.

Fig. 2 shows the trajectories of the missile (M) and the target (T) in the Inertia-frame, where, C1 and C2 are the collision points for the PSM guidance law and the SM guidance law, respectively. The interception time is 9.04 s for the PSM guidance law and 12.38 s for the SM guidance law.

Fig. 3 illustrates the guidance commands. Fig. 3(b) and (c) show that the PSM guidance law requires fewer efforts in tangential components of input vector, i.e., u_θ and u_ϕ, than those for the SM guidance law. Also, Fig. 3(a) demonstrates that the amount of changes of radial control effort for the PSM law is less than that of SM guidance law. In brief, the control energy ($\int_0^{t_f} u^T u dt$) of the PSM law is much u_r less than that of the SM law.

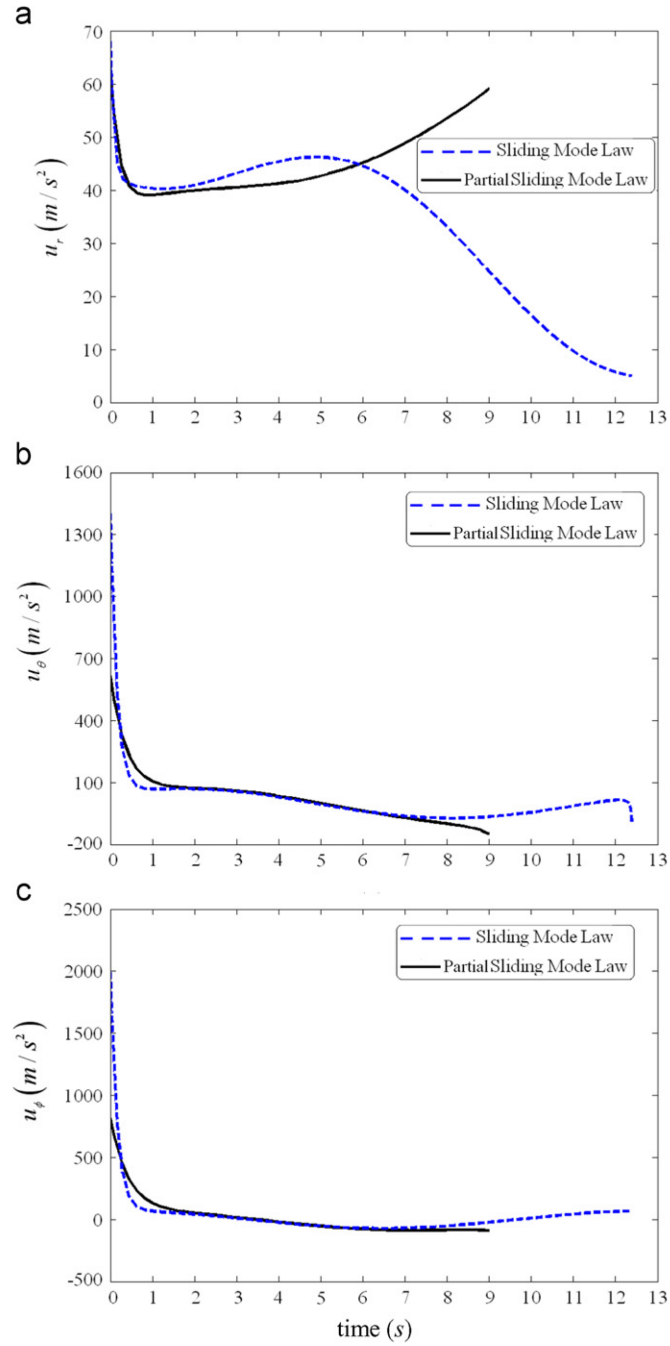

Fig. 3. (a) Radial guidance command (u_r), (b) tangential guidance command u_θ and (c) tangential guidance command u_ϕ.

5. Conclusion

In this paper a new viewpoint to the guidance problem was presented. It was demonstrated that in practical approach to guidance problem, the desirable behaviors of state variables are different with respect to each other, and the asymptotic convergence behavior is not ideal for all state variables. Therefore, based on PSM technique, a new robust nonlinear guidance law was developed and its effectiveness in interception of maneuvering targets was shown analytically and also through simulations. In the simulation section, complicated and highly maneuvering targets (sinusoidal targets) was considered and it was shown that the proposed method, in comparison to sliding mode method, can lead to better results. It is because this method (in contrast to other robust nonlinear guidance laws like SM guidance law) does not need to regulate $\dot{r}$ to a negative constant c. The value of this constant, considerably affects the performance of guidance law and in order to determine it efficiently, the maneuvering model of

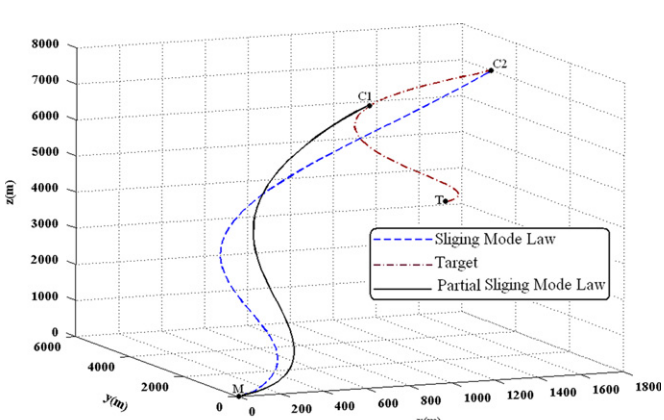

Fig. 2. Trajectories of the missile (M) and the target (T).

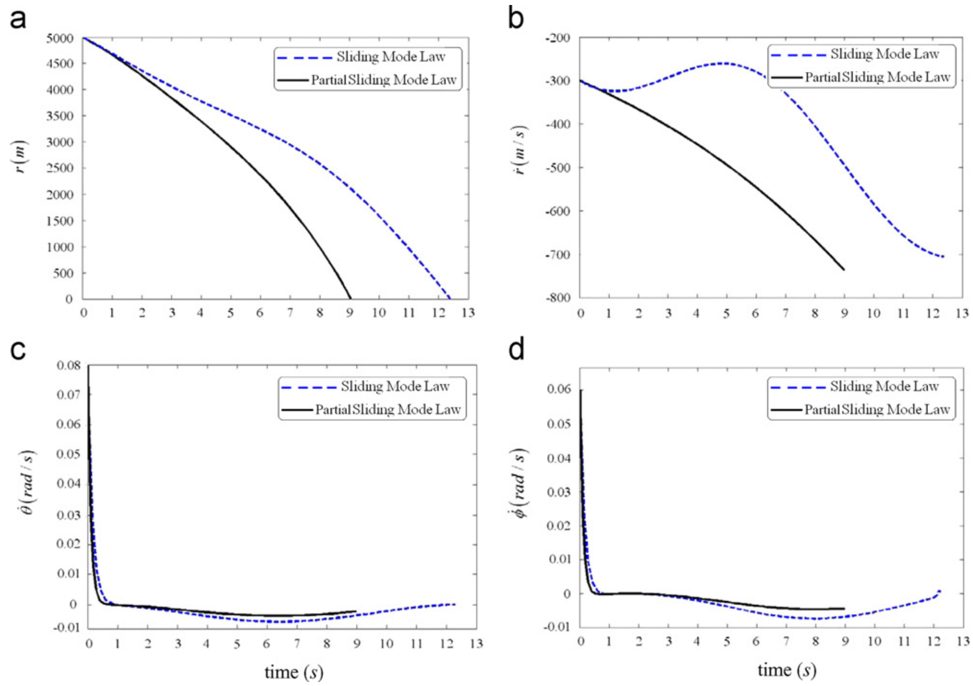

Fig. 4. (a) Relative distances between the missile and the target (r), (b) radial component of relative speed ($\dot{r}$), (c) pitch LOS-angular rate ($\dot{\theta}$) and (d) yaw LOS-angular rate ($\dot{\phi}$).

the target should be known a priori for the guidance system, while it is not always possible in practice.

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